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· Artificial variables Technique

— In L.P.P. some constraints may have the signs $\geq$ or $=$ with all b_i's +ve.

In such problems we introduce surplus variables in the constraints with sign $\geq$.

In these problems we can not get the starting basic matrix. So, to avoid this difficulty we add one more variable to each of such constraints. These variables are called "artificial variables".

These variables are fictitious and represent no physical entities.

Such problems may be solved by two methods.

Method I : — Two phase Method

Method II : — Big-M method.

Q. Solve the following L.P.P. by Big-M method.

$$\min z = 2x_1 + x_2$$

s.t.

$$3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \leq 3$$
$$x_1, x_2 \geq 0.$$

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Solution.

First we convert the problem of minimization to the maximization problem by taking the objective function as

$$z' = -z$$

$\max z' = -z = -2x_1 - x_2$

Introducing slack variable x_4, surplus variable x_3 and artificial variables A_1 and A_2 we get

$\max z' = -2x_1 - x_2 + 0.x_3 + 0.x_4 - MA_1 - MA_2$

s.t.

$$3x_1 + x_2 + A_1 = 3$$
$$4x_1 + 3x_2 - x_3 + A_2 = 6$$
$$x_1 + 2x_2 + x_4 = 3$$
$$x_1, x_2, x_3, x_4 \geq 0.$$

putting $x_1 = x_2 = x_3 = 0$, we get

$A_1 = 3$
$A_2 = 6$
$x_4 = 3$

$$\therefore X_B = \begin{bmatrix} A_1 \\ A_2 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix}$$

which is initial B.F.S.

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B	C_B	C_j / X_B	-2 / $Y_1(\alpha_1)$	-1 / $Y_2(\alpha_2)$	0 / $Y_3(\alpha_3)$	0 / $Y_4(\beta_3)$	-M / $A_1(\beta_1)$	-M / $A_2(\beta_2)$	min $\frac{X_B}{Y_1}$
A_1	-M	3	$\boxed{3}$	-1	0	0	1	0	$\frac{3}{3}=1 \rightarrow$ min
A_2	-M	6	4	3	-1	0	0	1	$\frac{6}{4}=\frac{3}{2}$
Y_4	0	3	1	2	0	1	0	0	3
$Z=C_BX_B$ $=-9M$		Δ_j	-2+7M ↑max.	-1+4M	-M	0	0	0	

$Z = C_B X_B = (-M,-M,0)(3,6,3) = -3M-6M+0 = -9M$

$\Delta_j = C_j - C_B Y_j$

$\Delta_1 = C_1 - C_B Y_1 = -2-(-M,-M,0)(3,4,1)$
$= -2-(-3M-4M+0)$
$= -2+7M$

$\Delta_2 = C_2 - C_B Y_2 = -1-(-M,-M,0)(1,3,2)$
$= -1-(-M-3M+0)$
$= -1-(-4M)$
$= 1+4M$

$\Delta_3 = C_3 - C_B Y_3 = 0-(-M,-M,0)(0,-1,0)$
$= 0-(0+M+0)$
$= -M$

Incoming vector $\alpha_j = \max.\{\Delta_j\}$
$= \max\{\Delta_1, \Delta_2, \Delta_3\}$
$= \max.\{-2+7M, -1+4M, -M\}$
put $M=1$
$= \max\{5, 3, -1\}$
$= 5 = -2+7M$
$= \Delta_1$
i.e. $j=1$

$\therefore \alpha_1 (Y_1)$ is incoming vector outgoing vector (β_r).

$$\frac{x_{Br}}{y_{rj}} = \min\left\{\frac{x_{Bi}}{y_{ij}},\ y_{ij}>0\right\}.$$

$$= \min\left\{\frac{x_{B1}}{y_{11}}, \frac{x_{B2}}{y_{21}}, \frac{x_{B3}}{y_{31}}\right\}$$

$$= \min\left\{\frac{3}{3}, \frac{6}{4}, \frac{3}{1}\right\}.$$

$$= \min\left\{1, \frac{3}{2}, 3\right\}$$

$$= 1$$

$$\frac{x_{Br}}{y_{rj}} = \frac{x_{B1}}{y_{11}}$$

$\Rightarrow r = 1$

$\therefore$ $\beta_1(A_1)$ is outgoing vector.

$R_1 \to \frac{1}{3} R_1$

$R_2 \to R_2 - 4 \cdot \frac{1}{3} R_1$, $R_3 \to R_3 - \frac{1}{3} R_1$

		c_j	-2	-1	0	0	$-M$	$-M$	
B	C_B	X_B	$Y_1(\beta_1)$	$Y_2(\alpha_2)$	$Y_3(\alpha_3)$	$Y_4(\beta_3)$	A_1	$A_2(\beta_2)$	$\frac{X_B}{Y_2}$
Y_1	-2	1	1	$1/3$	0	0		0	3
A_2	$-M$	2	0	$\boxed{5/3}$	-1	0		1	$6/5$
Y_4	0	2	0	$5/3$	0	1		0	$6/5$
$Z = C_B X_B = -2-2M$		Δ_j	0	$-1+\frac{(2+5M)}{3}$	$-M$	0		0	

$\uparrow$ max.

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$Z = C_B X_B = (-2, -M, 0)(1, 2, 2)$
$= -2 - 2M$

$\Delta_j = C_j - C_B Y_j$

$\Delta_1 = C_1 - C_B Y_1 = -2 - (-2, -M, 0)(1, 0, 0)$

$\Delta_2 = C_2 - C_B Y_2 = -1 - (-2, -M, 0)\left(\frac{1}{3}, \frac{5}{3}, \frac{5}{3}\right)$
$= -1 - \left(-\frac{2}{3} - \frac{5M}{3} + 0\right)$
$= -1 + \left(\frac{2+5M}{3}\right)$

$\Delta_3 = C_3 - C_B Y_3 = 0 - (-2, -M, 0)(0, -1, 0)$
$= -(\cancel{0} + M)$
$= -M$

Incoming vector $(\alpha_j) = \max \Delta_j$
$= \max\{\Delta_2, \Delta_3\}$
$= \max\left\{-1 + \left(\frac{2+5M}{3}\right), -M\right\}$
put $M = 1$
$= \max\left\{-1 + \frac{7}{3}, -1\right\}$
$= \max\left\{\frac{4}{3}, -1\right\}$
$= \frac{4}{3} = -1 + \left(\frac{2+5M}{3}\right)$
$= \Delta_2$

$\Rightarrow j = 2$

i.e. $\alpha_2 (Y_2)$ is incoming vector outgoing vector (B_r)

$$\frac{x_{Br}}{y_{rj}} = \min\left\{\frac{x_{Bi}}{y_{ij}},\ y_{ij} > 0\right\}$$

$$= \min\left\{\frac{x_{B1}}{y_{12}}, \frac{x_{B2}}{y_{22}}, \frac{x_{B3}}{y_{32}}\right\}$$

$$= \min\left\{\frac{1}{1/3}, \frac{2}{5/3}, \frac{2}{5/3}\right\}.$$

$$= \min\left\{3, \frac{6}{5}, \frac{6}{5}\right\}.$$

$$= \frac{6}{5}$$

$$\frac{x_{Br}}{y_{rj}} = \frac{x_{B2}}{y_{22}} \text{ or } \frac{x_{B3}}{y_{32}}$$

$\Rightarrow r = 2$ or 3.

let $r = 2$

$\therefore$ $B_2 (A_2)$ is outgoing vector

		C_j	-2	-1	0	0		
B	C_B	X_B	Y_1	Y_2	Y_3	Y_4	A_1	A_2
Y_1	-2	3/5	1	0	1/5	0		
Y_2	-1	6/5	0	1	-3/5	0		
Y_4	0	0	0	0	1	1		
$Z' = -\frac{12}{5}$		Δ_j	0	0	-1/5	0		

since all $\Delta_j \le 0$
the solution obtained above is optimal

$\therefore$ $\max z' = -\frac{12}{5}$ $\Rightarrow$ $\min z = \frac{12}{5}$

$x_1 = 3/5$ Ans.

$x_2 = 6/5$